

B-sc. II (H) - PAPER - IIIA.

Entropy and entropy change of Ideal gas.

Entropy - Entropy of a system is an extensive property which provides a measure of disorder or randomness. It is denoted by S . It is a state function and not measured directly but only change in entropy of the system is measured.

Mathematically change in entropy of a reversible system is defined as

$$\Delta S = \frac{q_{rev}}{T}$$

Where q_{rev} is change in heat of system reversibly at temperature T . Entropy unit is $\text{JK}^{-1}\text{mol}^{-1}$

If system changes from A to B reversibly then entropy change is given as $\Delta S_{\text{sys}} = \frac{q_{rev}}{T}$ and corresponding

entropy change in surroundings $\Delta S_{\text{surr}} = -\frac{q_{rev}}{T}$

for reversible process.

$$\Delta S_{\text{system}} + \Delta S_{\text{surround}} = 0$$

Irreversible process $\Delta S_{\text{system}} + \Delta S_{\text{surround}} > 0$

As all the natural processes are spontaneous i.e. irreversible, the entropy of Universe is always increasing.

Entropy change in an ideal gas :-

The change in entropy of system is defined as

$$ds = \frac{dq_{rev}}{T} \quad \text{--- (i)}$$

$$\text{or, } dq_{rev} = T ds$$

From First law of Thermodynamics

$$dq_{rev} = dE + PdV \quad \text{--- (ii)}$$

or From equation (i) and (ii)

$$ds = \frac{dE + PdV}{T}$$

$$= \frac{dE}{T} + \frac{PdV}{T} \quad \text{--- (iii)}$$

We know $c_v = \frac{dE}{dT}$ or $dE = c_v \cdot dT$

$$\therefore ds = c_v \cdot \frac{dT}{T} + \frac{PdV}{T} \quad \text{--- (iv)}$$

For one mole of gas $PV = RT$ or $P = \frac{RT}{V}$ --- (v)
From equation (iv) and (v)

$$ds = c_v \cdot \frac{dT}{T} + \frac{RdV}{V} \quad \text{--- (vi)}$$

on Integrating we get

$$s = c_v \ln T + R \ln V + \text{const } S_0 \quad \text{--- (vii)}$$

$S_0 = \text{constant.}$

For initial state (1)

$$s_1 = c_v \ln T_1 + R \ln V_1 + S_0$$

or final state (2)

$$s_2 = c_v \ln T_2 + R \ln V_2 + S_0$$

or,

$$\Delta S = S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

(viii)

This is required relation between change of entropy of ideal gas when temperature and volume of initial state and final state is known.

Alternative form of equation (iii)

$$C_p - C_v = R \text{ or } C_v = C_p - R \text{ --- (ix)}$$

$$\text{and } PV = RT \text{ and } V = \frac{RT}{P} \text{ --- (x)}$$

Putting value of eq (ix) and (x) in eq (vii) we get.

$$S = (C_p - R) \ln T + R \ln \frac{RT}{P} + S_0$$

$$S = C_p \ln T - R \ln T + R \ln R + R \ln T - R \ln P + S_0$$

$$\text{or } S = C_p \ln T - R \ln P + (R \ln R + S_0)$$

$$\text{or } S = C_p \ln T - R \ln P + S_0' \text{ --- (xi)}$$

S_0' = another ~~equation~~ constant

Then entropy change can be written

$$\Delta S = S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$